Introduction

In 1990 two of us (Karl Schaffer and Erik Stern) created our first math dance stage performance, *Dr. Schaffer and Mr. Stern: Two Guys Dancing about Math.* Since then the members of our dance company have developed five more math dance shows, performed for over a hundred thousand people, and taught workshops to hundreds of classes. Many teachers have written or spoken to us about the strengths and weaknesses of the particular activities and have helped us hone our methods. The ages and grade levels of participants in our workshops have run the gamut from kindergarten to college level math and dance classes, from teacher workshops to gatherings of artists and scientists.

We have been collecting, writing and discussing the activities in this book since 1991 and are excited to offer them *now* to teachers. We have included the activities which we tend to teach the most. Chapters are organized largely by mathematical concepts, and somewhat by dance categories. Chapters 1–3 involve counting in some fashion, chapter 4 is spatial in focus and serves as a segue into the rest of the chapters, which address geometry in a variety of ways. Chapters 9 through 12 involve an easily constructed prop. We hope this organization for the book will help teachers choose the appropriate exercise to fit the mathematics they are working on.

We are not proposing that these activities replace traditional math or traditional dance instruction — though that is a provocative idea for the future! This book is designed for those crucial junctures when a concept needs to be understood mentally, physically and emotionally. Or for the infusion of energy and excitement that classes sometimes need. Or to reach those students whose learning styles are not compatible with traditional methods of teaching. Or for teachers and students who want to experience dance and mathematics in a new way.

Once the concepts we have introduced are felt and thought, educators can return to the subjects with renewed vigor, and study individual areas in more depth.

Why Math and Dance?

At first glance it might seem that mathematics, that realm of rationality, and dance, that art of physical and emotional expression, have little in common. In our own experience as dancers and mathematicians, however, the two subjects are inextricably linked. When we choreograph a new dance or investigate a mathematical problem we are doing much the same thing: creatively exploring patterns in space and time with an eye toward aesthetic potential.

As we have shared our perspective in teacher workshops, we have witnessed spontaneous and inadvertent acts that confirm our belief that, rather than being separate educational areas, the physical and the mental belong together. After a performance of our basketball dance at a school in Santa Cruz County, for instance, a boy who looked to be in first or second grade ran after us and said, "You know, I was just playing basketball and I realized that I was dancing!"

Sometimes the students, or we ourselves, have been surprised by what happens. At an elementary school in Hawai'i, when we announced to a group of fifth graders that we were going to do math and dance, students in the class pointed to one girl and said, "She's good at math!" As the workshop progressed, other students excelled. One boy was extremely adept at the rhythmic exercises. Another girl was adept with rotational symmetry. The class was very surprised and began to see that there are many examples of mathematical thinking.

Sometimes schools have taken our work farther than we anticipated. At the Alabama High School of Fine Arts in Birmingham, a group of seniors created a performance with giant tangrams (chapters 11 & 12) which they used to recruit new students into the school. They then taught the performance to a group from the next year's graduating class.

Though the sentiment has been expressed in many ways, one school in northern Nevada stands out. A teacher said, "I wish my desire to move had been used as a tool to learn math when I was a kid."

Our work in educational settings, from the most elaborately funded private schools to some of the least supported public schools, has demonstrated that teachers who are willing to

engage students with alternative methods are often quite successful. These methods allow for a variety of learning styles and cultural backgrounds, and also keep that spark of curiosity alive.

To us, the question is not "Why combine mathematics and dance, the creative and the cognitive, in class?" but rather "Why were they ever separated in the first place?"

Creating our First Show

In 1989, after having collaborated for three years on dances, two of us (Karl Schaffer and Erik Stern) began talking about the similarities between dance and mathematics.

Our discussions covered a wide range of topics. We noticed that math and dance both deal with codified concepts, such as symmetry, spatial awareness, counting problems and patterns. We also noticed aesthetic similarities: the need for internal consistency, the goal of striking a balance between analysis and intuition, and how either one could be abstract as well as worldly.

You could even sweat doing both, we realized.

These nascent ideas led us to create a performance designed for students, educators and families. As we worked on the show, we became convinced of its value; we also began to see how difficult it was going to be to make our vision clear and accessible. As teachers or professionals who cross established boundaries know, building a bridge between disciplines that is easily traversed is a daunting task.

We contacted the California Math Council-North and inquired if our performance might be appropriate for their next conference. Sherry Fraser, then program chair, said, "Well, can I see it?" She drove to Santa Cruz to watch the work in progress. After discussing our approach with us, she booked us as the opening event for the 1990 conference at Asilomar. Sherry's intelligence and enthusiasm gave us encouragement and feedback we needed.

At that time mathematics educators and organizations were re-evaluating not only how mathematics was taught, but how it was viewed. Rapid development of information technology fueled change in mathematics as well. Calculators that freed students from the constant drudgery of manual computation led to new demands that mathematics be exploratory and involve higher level thinking. Efforts to develop artificial intelligence came side-by-side with new understandings of how human intelligence and learning really work, leading to demands for multiple approaches in teaching math. And mathematics was changing itself as new areas such as chaos theory and fractals joined the discrete math topics more relevant to a world of computers.

Besides being professional dancers, we had the backgrounds which allowed us to look into the science side of things: Karl Schaffer was and is a mathematics professor and Erik Stern's undergraduate degree is in biology. Our own experiences in education had led us to see that other approaches in the sciences could be valuable. The then recently published National Council of Teachers of Mathematics (NCTM) Standards confirmed our feelings that concepts such as estimation and number sense, symmetry and spatial thinking, were as relevant to math education as the operations of arithmetic.

We also drew on the numerous performances we had seen and participated in that sought to link the arts with the curriculum. However, when it came to dance and science, we felt that little had been done, and what had been done skimmed the surface. The performances we saw were either more committed to the art or more committed to the science, but did not seem to have a firm foundation in both. When we did find efforts to use movement with mathematics they were usually centered on the pre-kindergarten and primary grades. Our extension of movement/mathematics activities to secondary and college levels has remained a hallmark of our work.

We were inspired to try out our own ideas by the numerous choreographers, composers, visual artists and playwrights who have addressed mathematics, either by imbedding the ideas of mathematics into the art itself, or by treating the subject more topically. The more we delved into math dance, the more we encountered performers, mathematicians and enthusiasts who were also exploring these links.

As with these other artists, our hopes for the project extended beyond an interest in sharing with audiences mathematical concepts. The project was a great opportunity to do for dance what mathematics education reformers wanted to do for math; that is, to make dance accessible, show its presence in everything we do, and expand the ways it is typically taught.

To us, dance and mathematics were more than equals, they were manifestations of the same interest in aesthetics and form, thought and expression. Presenting them in this light, we hoped, would also convey that the vast divide between the arts and sciences, one which includes funding and value differences, was a divide borne of misconceptions. The perception of dance and

mathematics (and, by implication, art and science) as incompatible did not reflect the nature of these two essential endeavors — at least as we experienced them.

We knew, as all people involved in the arts know, that mathematics will always be required, but the arts are ever on precarious ground. The arts in America are too often left to the individual teacher, principal, school or district and are rarely given the imprimatur of national attention and funding. Mathematics would always be taught; dance would not. We wanted to share our beliefs in the strength and compatibility of these subjects.

Interestingly enough, dance had gone through a similar historical process as math education. The innovations of the 60's and 70's sought dance in the everyday, and also led to the question "What is dance?" But the results of these rarefied investigations did not trickle down to public schools.

Our first step in creating the performance was to look at the dances we had already made or were in the process of making, in terms of their appeal from both a choreographic and mathematical point of view.

The first dance we looked at was called "Private Fly," in which two guys in trench coats dance with fly swatters. In addition to types of symmetries we normally think of, such as reflection and rotation, the fly swatters got progressively larger, which introduced the symmetry of scale. The idea of symmetry is an underpinning of dance, though we later found that the different types of symmetry are rarely discussed in as explicit manner in choreographic classes as in mathematics classes.

Moreover, symmetry was also being reintroduced in math education by way of geometry. Fortunately for spatially and kinesthetically oriented students, this re-introduction of symmetry had students making, touching, drawing and visualizing geometric objects.

Other dances included in this first concert relating dance and mathematics:

- A tap dance, which dealt with patterns and some of the arithmetic associated with those patterns, and led to an audience interaction with interwoven rhythms.
- An almost vaudevillian hand-shake routine, which served to introduce the characters, and led to a discussion of estimation and number sense, as well as a workshop on counting handshakes.
- "Rotation," a trio for two dancers and a basketball, which led to a section in the performance about the physics of motion.

Our choreography and movement material showed that dance can arise from everyday activities (playing basketball, shaking hands, and even swatting flies). That we were taking stereotypical guy activities like sports and showing that they could be dance, broke the frequent stereotypes about dance. During *Two Guys Dancing About Math* we also tell the story of the mathematical work of Ada Lovelace, a 19th century mathematician who surmounted the barriers she faced.

It is hardly a secret that many people, including teachers, do not have a positive reaction to math class. The particular reasons may vary: a traumatic experience with math in the past, an unsupportive teacher, the symbols seem confusing, no allowance to work out problems in a way that is comfortable, acceptance of popular myths about

who can succeed at math, or even a lack of aptitude (though this last one seems more rare).

The upshot is the same: students who need to learn mathematics are not coming away with the skills and knowledge required of them. Of equal importance, or maybe the root cause of the problem, is that students weak in math are not confident and enthused about the subject.

We were acutely aware of these responses to mathematics and interested in incorporating them into our performance. As a result, two rather opposite characters surfaced: one was analytic, Dr. Schaffer; the other, instinctive, Mr. Stern.

We wanted to show that there are many ways to do and experience mathematics and dance. We wanted students and teachers to get a sense of the different ways people can learn and express their love of various subjects.

When we premiered *Dr. Schaffer and Mr. Stern: Two Guys Dancing About Math* at the 1990 Asilomar Conference, the idea that our fascination would be of the same interest to others seemed a little unlikely. But, as it turns out, it has crossed the 400 performances and ten-year mark and is still in demand.

When we showed up at schools, kids, and sometimes even teachers, said things like "What are you gonna do, multiply with your bodies?" Our reply was usually "Just wait and see the show."

Afterwards, these questions made us want to clarify for ourselves the conceptual basis of what we were doing. In our show, dance was not a kind of mnemonic device. It was not the dance version of "Thirty days hath September, April, June and November…" Nor were we only using the body as a way of understanding concepts, and then discarding the expression inherent in the act of moving. The use of manipulatives has long been a part of education, and recently has experienced a much needed revival; however, the body, to us, is more than a big "manipulative." Its use can be an end in itself; moreover, the attention to aesthetics and perceptions that dance leads to is of equal value to the introduction of concepts.

We were looking to exploit the most basic ideas that underlie dance and mathematics, and, though those underlying ideas may seem absurdly simple, we felt they could lead to very clear and interesting problems both in the realm of art and science.

At its most complex and abstract, mathematics appears to be very far from dance or anything physical and palpable. Its concepts and symbols are manipulated by rules that sometimes appear mysterious; its purpose seems to be for the enjoyment or productiveness of those who practice it (and for the pain of many others).

But what is mathematics at its most basic level? Quantity, size, relationship, shape and space, reasoning, discerning patterns, representing a concept by a word or a symbol. And even before these ideas come abilities which could be called "pre-mathematical" thinking skills.

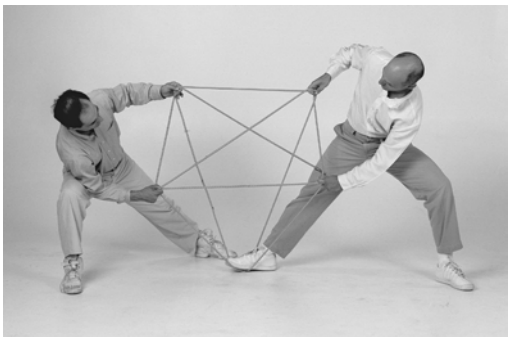
Included in the pre-mathematical category of concepts are bigger and smaller, same and different, before and after, including and excluding, inside and outside; and also remembering where you are and where you are going. We believe that the understanding of mathematics begins in the physical realm with these concepts.

To our way of thinking, this list of pre-mathematical skills is very close to pre-dancical (to coin a word) skills. A young dancer also needs to know before and after, same and different, and so on. These essential skills are the predecessors to dance and mathematics, and constitute the essential overlap of the disciplines.

From the Stage to the Classroom

Schools continued requesting that we get children and teachers doing what we were doing on stage; moving and thinking, thinking and moving. The first workshops we offered were extensions of the ideas explored in our concert, adapted to be safe and accessible. Later, after collaborating with Scott Kim on performances and workshops, our creative process sometimes went the other way: a workshop idea might lead to a new dance.

In all cases, the exercises were designed to be dualistic in the best sense. Ideally, students and teachers should be able to slip naturally from the exercises into significant mathematical explorations; they also should be able to take the exercises and choreograph studies which they could then perform for each other, the rest of the school, the P.T.A., or even the public.



Math Dance

Our work in the classroom has shown us that the ideas in mathematics are interesting to most everyone. The methods by which math is taught, however, can be a barrier. The interactive, visible, communicative properties of dance/movement can help bring mathematics to a comprehensible level.

Similarly, our work has shown us that the pure human energy of dance is universally interesting; the impulse is in everyone. But when the dancing is abstruse, people worry; when they feel they have to know the language of dance, that can also present a barrier.

Whole body, expressive, mathematical activities introduce new elements into the classroom that go beyond traditional manipulatives. Here is a list of some of the attributes of what we like to call ***math dance***:

Everyone participates. For example, one chapter has a trio of students create and explore three-fold rotationally symmetric shapes with their entire bodies. The exercise demands that everyone experience and demonstrate the mathematical concepts being introduced.

A deeper spatial thinking challenge. With manipulatives we can see what we are creating. With the entire body, we seem to operate from a three-dimensional picture in our minds. We suspect that using the entire body requires different (and at times more difficult) spatial thinking skills.

Kinesthetic learners. Individuals who learn kinesthetically find an experiential base to help them understand. Those who are less kinesthetically inclined get experiences that are often missing the higher one goes in the educational system.

Engages students. Involving one's body is exciting and memorable. It excites student interest.

New approach. The size of the product and the contrast between kinesthetic and, for example, symbolic mathematics, pushes students and teachers towards new insights and questions.

Communication. The scale of the exploration is expressive, and therefore allows students to experience mathematics as art and communication, and vice versa. In many cultures, arithmetic, spatial thinking and other mathematical skills are taught through stories, songs, games, and dances. The idea of mathematics as communication stimulates an interest in math that is broader, and forces students to develop their mathematical (as well as artistic) communication skills.

Aesthetics. Art deals with aesthetics; mathematicians feel that math is intrinsically beautiful and has other aesthetic qualities as well. Observing, responding to what is observed, and articulating that response (sometimes labeled "aesthetic valuing"), are skills that must be developed. "What seems better and why?" is a question which is an integral part of creating dances, a question which is posed, in various forms, throughout this book.

Helps students cross barriers. Introduces students to new areas; those who are unfamiliar with dance are given a route into this performing art. Those who are shy of math are similarly helped to find a way to experience it. For example, a unit which involves the manipulation of giant tangrams, where each person carries only one of the seven shapes, provides an excellent way to explore geometric principles and also can lead to story-telling, art, and dance.

Social event. The group and kinesthetic aspect of this work makes it public, and involves students in social interactions. Discussion and group work grow naturally out of the explorations. Dance becomes a less solitary venture, and mathematics becomes a clear intellectual, physical and social event to which teachers can refer.

Culture. The burgeoning field of ethnomathematics has drawn attention to how much mathematical thinking is present in all aspects of culture. Dance and mathematical thinking are found in all cultures, and we have tried to point to particular forms of dance that deal with mathematical ideas in interesting ways. This kind of work can lead to new respect for diverse cultures, and help integrate learning about culture into the classroom.

Expands each discipline. As we began to find our way teaching workshops, we noticed that movement was encouraging experimentation. It was pushing people away from inflexibility and toward play and interaction, away from a rigid search for right answers that is often associated with mathematics and toward the sort of play and interaction that mathematicians experience. Similarly the work shows the connection dance has to ideas and forms of analysis that are not always associated with the discipline.

For specific directions on how to use this book and helpful preparatory information, please read the next chapter *How To Use This Book.*